## **First-Order Differential Equations**

## Differential Equations X. Du

- First-Order Ordinary Differential Equations take the from  $\frac{dy}{dx} = f(x, y)$
- Slope fields plots the behavior (slope) at certain points using a differential equation
  - **Isoclines** curves along which the slope is constant. Common method for drawing slope fields
- Integral curve
  - Curve that represents a specific solution of an ordinary differential equation.
  - Tangent to slope field lines.
  - Cannot cross each other as long as  $\frac{dy}{dx} = f(x, y)$  and  $\frac{\partial f}{\partial y}$  are continuous
- Steady State an asymptote that all solutions of the differential equation tend to regardless of initial condition. In solutions to most differential equations, they do not exist.
- Existence Theorem: If  $\frac{dy}{dx} = f(x, y)$  is continuous on an open disk containing  $(x_o, y_o)$ ,

then there exists a solution to  $\frac{dy}{dx} = f(x, y) \operatorname{at}(x_o, y_o)$ .

• Corollary (by contraposition): If there is not a solution to  $\frac{dy}{dx} = f(x, y)$  at  $(x_o, y_o)$ ,

then  $\frac{dy}{dx} = f(x, y)$  is discontinuous on an open disk containing  $(x_o, y_o)$ .

- Be careful! Inverse and converse are not true!
- Uniqueness Theorem: If  $\frac{dy}{dx} = f(x, y)$  and  $\frac{\partial f}{\partial y}$  are continuous on an open disk containing

 $(x_o, y_o)$ , then there exists one and only one solution to  $\frac{dy}{dx} = f(x, y)$  at  $(x_o, y_o)$ .

• Corollary (by contraposition): If there is not exactly one solution to  $\frac{dy}{dx} = f(x, y)$ 

at 
$$(x_o, y_o)$$
, then either  $\frac{dy}{dx} = f(x, y)$  or  $\frac{\partial f}{\partial y}$  (or both  $\frac{dy}{dx} = f(x, y)$  and  $\frac{\partial f}{\partial y}$ ) is

discontinuous on an open disk containing  $(x_o, y_o)$ .

- Be careful! Inverse and converse are not true!
- Applications:
  - o Exponential growth and decay models
  - Projectile motion with drag
  - RC circuits and RL circuits
  - Mixing and diffusion